Hydrological Challenges and Human Impacts in the Great Lakes

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Abstract The Great Lakes system is inherently dynamic, with water levels and flows subject to natural variations and human influences. Seasonal weather patterns, climate change, and human activities such as water withdrawal and regulation of outflows through dams and locks significantly impact the system's hydrology. These factors together contribute to fluctuating water levels, which pose challenges for shoreline management, ecological balance, and water resource allocation. To better utilize water resources and satisfy the demands of various stakeholders, several models have been established: Model I: Scoreboard and Genetic Algorithm; Model II: Netflow Model; Model III: Multi-target Optimization with Ford-Fulkerson Algorithm. Finally, sensitivity analysis of the model is conducted, demonstrating the model's strong adaptability and ability to address diverse requirements.

Keywords Genetic Algorithm, Multi-objective Optimizing Model, AHP, Netflow Model

1. Introduction

1.1 Problem Background

The Great Lakes, comprising Lakes Superior, Michigan, Huron, Erie, and Ontario, constitute one of the largest freshwater systems on Earth. They sustain diverse ecosystems, economies, and cultures. However, this critical resource faces substantial environmental management challenges. Water quality issues, such as harmful algal blooms and contaminated drinking water sources, arise from pollution caused by industrial discharges, urban stormwater, and agricultural runoff. Invasive species introduced via ballast water from ocean-going ships have disrupted natural ecosystems and impacted commercial and recreational fisheries essential to the local economy. Climate change further complicates water management by altering precipitation patterns, increasing evaporation rates, and causing variable water levels that threaten coastal habitats, infrastructure, and water availability.

Figure 1. The Beauty of Great Lakes

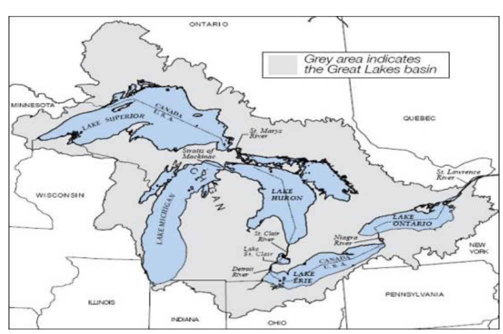

Figure 2. Map of the Great Lakes

These images are sourced from the United States Geological Survey (USGS). To address these challenges, scientifically grounded and cooperative management approaches are needed to balance water supply, ecological health, and

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commercial interests across state lines. Coordinated strategies are essential to ensure the sustainability of this vital freshwater resource for future generations.

1.2 Literature Review

The main focus problem is to control the water level to satisfy the demands of each party and achieve the relative steady of the optimal water level. Brain P. Neff and J. R. Nicolas created an approach to quantify the inflow and outflow of different lakes and serval factors may affect the net flow, which will change the water level ultimately.

$$\text{Outflows} = CC_{out} + E + D_{out} + C_{use} \quad (1)$$

Where CC_{out} is connecting-channel outflow; E is evaporation from the lake surface; D_{out} is diversion away from the Great Lakes; and C_{use} is consumptive use of Great Lakes Water.

For the inflow, the authors mention that the **greatest component** is over-lake precipitation. Then, the authors also point out that the runoff, which includes streamflow and direct overland flow to the Great Lakes, should also be considered. Streamflow should be calculated through gagged areas and ungagged areas separately. For gaged areas, it is critical to consider the multiple stages or velocities. For ungagged areas, adding a multiplier to the data from gaged areas would better adjust the suitability of the measurement of ungagged areas, which also solves the problem of incontinuity [1]. After offsetting the inflow and outflow, it is easy to find the net flow for each lake in each month and can better assist the team deciding the timing of open or close the gate of the dams.

$$\text{Inflows} = P + R + GW_{in} + D_{in} + CC_{in} \quad (2)$$

Where P is precipitation falling directly on the lake, R is runoff, GW_{in} is net discharge groundwater inflow, D_{in} is diversion into the Great Lakes, and CC_{in} is connecting-channel inflow.

Another author, Parker A. Norton, not only considers the above factors but also considers the trend analysis of climate change in the Great Lakes. By applying the time series model with lags, Norton could predict the precipitation and evaporation in the next months in a reasonable way. This would better explain how the trend in climate change would affect the water level of the Great Lakes. Since it is a kind of historical simulation, Norton holds the assumption that history would repeat in the future and emphasizes he importance of **serial correlation** [2].

1.3 My Work

This study quantifies and defines stakeholder demands, sets water level limits, and divides the year into peak and non-peak seasons. By integrating constraint functions with a genetic algorithm, the optimal water level for Lake Ontario is calculated.

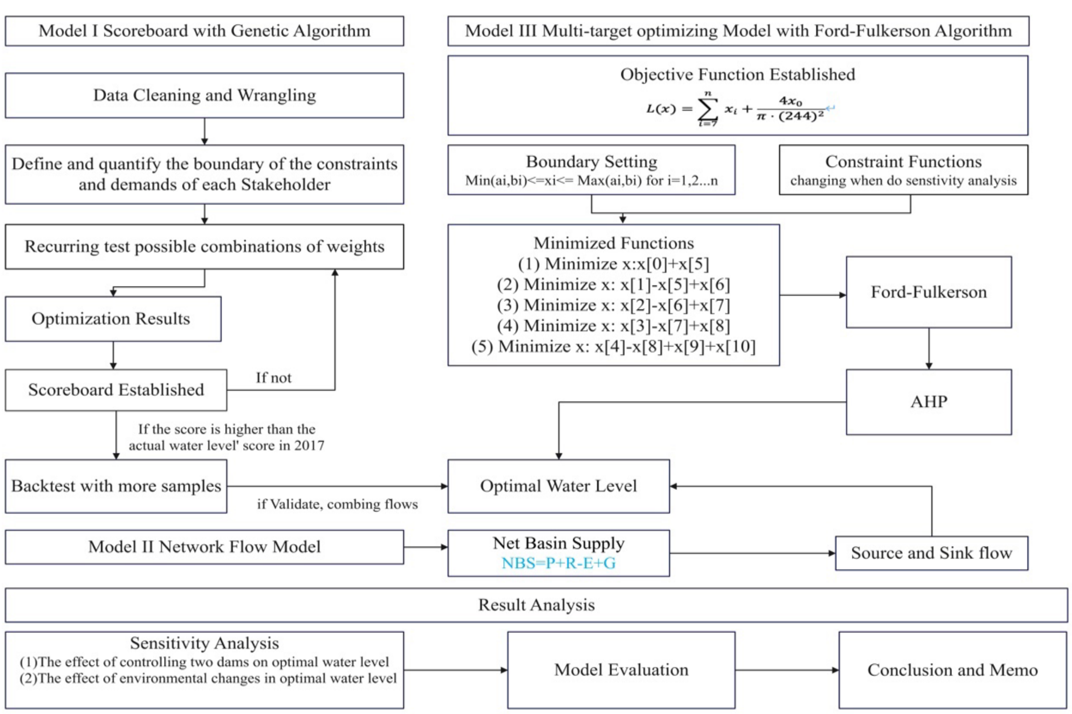

Figure 3. Main idea

2. Assumption and Model Overview

The water level of the Great Lakes is affected by many factors and if I consider all factors, it is possible to causes overfitting problems when I optimize the outputs. Therefore, I decide to ignore some minor variables and assume the constant rate of some variables, such as the inflow of the ground water. Because this kind of variable would not be affected by the seasons.

Assumption 1: Precipitation distributes uniformly across the entire surface of the lake.

Justification: This assumption allows for simplified calculations by not requiring detailed spatial data on precipitation.

Assumption 2: The rate of evaporation is constant throughout the calculation period.

Justification: It simplifies the model by using a uniform evaporation rate, but it might not capture the true variability of evaporation that occurs due to seasonal or diurnal changes.

Assumption 3: The amount of surface runoff into the lake remains stable during the calculation period.

Justification: This approach simplifies the input of water from surface runoff by not accounting for fluctuations due to extreme weather events or seasonal variations, making hydrological calculations more straightforward.

Assumption 4: Groundwater flow into or out of the lake is a constant value.

Justification: By treating groundwater flow as a constant, the model avoids the complexities of groundwater-surface water interactions.

Assumption 5: The boundaries of the lake are considered to be clear and constant, without any changes due to fluctuating water levels.

Justification: This assumption makes area and volume calculations more straightforward by not taking into account the potential expansion or contraction of the lake's surface area with changing water levels.

3 Model Preparation

3.1 Notation

Table 1. Notations

Symbols	Description
P	Monthly Precipitation directly on the lake
R	Monthly Runoff
GW_{in}	Monthly net discharged groundwater
n	Number of input layer neuron nodes
q	Number of neuron nodes in the output layer
μ	Multiple constants
x_0	Comparison Sequence
E	Monthly Evaporation
r_i	Grey correlation degree

Table 2. Other Symbols

Symbol	Description
MAD	Mean absolute Deviation
RMSE	Root Mean Squared error
MSE	Mean Squared error

3.2 Data Overview

Hydrological data were collected from NOAA, USGS, and Environment Canada. The dataset includes monthly measurements of precipitation, surface runoff, evaporation, and estimated groundwater flow across the Great Lakes basin. Precipitation and evaporation rates were sourced from weather stations and satellite observations, while runoff data were derived from river flow records. Groundwater contributions were estimated based on regional hydrogeological studies [5].

Table 3. Data Collection

Database	Data Sources	Data Type
Precipitation	NOAA - Great Lakes Environmental Research Laboratory	Geography
Evaporation	NOAA - Great Lakes Environmental Research Laboratory	Geography
Runoff	NOAA - Great Lakes Environmental Research Laboratory	Greography

3.2 Data Processing

Data underwent rigorous cleaning, including handling outliers and missing values via interpolation and temporal averaging. All precipitation and evaporation data were standardized to cubic meters per second, accounting for lake surface area and time duration.

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Groundwater discharge to the Great Lakes, often overlooked, was considered. Although large-scale measurement is challenging, preliminary estimates suggest groundwater inputs are not a significant component of the water balance [3].

4 Model I: Scoreboard and Genetic Algorithm

4.1 Model Establishment

4.1.1 Quantify the demands

Water levels in Lake Ontario were categorized as follows:

- High: >70% of historical average
- Low: <30% of historical average
- Extreme high: >85%
- Extreme low: <15%

Peak electricity demand months (May–September) were identified for water level optimization.

4.1.2 Scoreboard Mechanism and Algorithm Application

A genetic algorithm (GA) via the DEAP library was used to dynamically adjust water levels. The GA evolves a population of 12-month water level sequences through selection, crossover, and mutation, evaluated by a custom fitness function incorporating stakeholder preferences and penalties.

4.2 Test the Model

The initial population (P) consists of randomly generated individuals, each representing a water level scenario for each month of the year. Assuming that there are individuals, the initial population can be expressed as:

$$P=\{I_1,I_2,\ldots,I_N\}$$

Where each individual I_N is a 12-dimensional vector representing the water level for 12 months:

$$I_i=[l_1,l_2,\ldots,l_{12}]$$

The fitness function (F) is used to evaluate the strengths and weaknesses of an individual. Specifically for this problem, the fitness function takes into account the extent to which the water level scenario meets the needs of each party and the penalty for abnormal water levels. For the individual I_i, its adaptation degree can be expressed as:

$$F(I_i)=\sum_{j=1}^{12}S(l_j,M_j)+\sum_{j=1}^{12}P(l_j)$$

Where $S(l_j,M_j)$ is a score based on how well the water level l_j in month M_j matches the criteria for the desired water level (including the needs of shipping companies, environmentalists, etc.). $P(l_j)$ is a penalty function applied to the water level l_j in month M_j for exceeding normal or extreme water level limits.

The selection process (S) is based on fitness and selects individuals from the current population to form the next generation. Methods such as Tournament Selection are often used:

$$S(P)\to P'$$

Crossover (C) is an operation in which selected individuals exchange some of their genes (in this case, the water level of the month) with a certain probability p_c:

$$C(I_a,I_b)\to(I_{a'},I_{b'})$$

Where I_a,I_b are new individuals generated by the crossover operation.

The mutation operation (M) introduces new genetic diversity by randomly changing part of an individual's genes with a certain probability p_m:

$$M(I_i)\to I_{i'}$$

Generate a new population P' from the current population P through selection, crossover, and mutation operations, and repeat the process until termination conditions are met (e.g. maximum number of generations is reached or fitness reaches a predetermined threshold).

Adaptation Calculation: calculates an adaptation score for each individual (i.e., a one-year water level scenario) based on how well each month's water level matches the ideal water level criterion and its corresponding penalty. Includes consideration of the negative effects of static stability (standard deviation) and consistency (mean absolute deviation, MAD):

$$F(I_i)=\sum_{j=1}^{12}S(l_j,M_j)-\text{std\_dev}(I_i)-\text{MA}D(I_i)+\sum_{j=1}^{12}P(l_j)$$

Where $S(l_j,M_j)$ calculates the score based on the demand corresponding to M_j and water level l_j and $P(l_j)$ calculates the penalty value based on the penalty function applied to water level l_j.

Through the above steps, the genetic algorithm is able to **search for the best or near-optimal solution** among multiple possible water level solutions, realizing the best solution in meeting environmental protection, navigation, and water quality. This model realizes the optimization of water level management while meeting the needs of environmental protection, shipping, residential life and energy generation.

Constraints Function Established:

Considering the needs of different stakeholders, I define mathematical models to minimize the overall cost or penalty. Here the score function and penalty function are defined in detail and the mathematical expression of the optimization problem is given:

For the water level l_j in M_j, the score function gives a score based on whether the water level meets the stakeholders' needs. This is specified as follows:

Shipping companies: want high and stable water levels

$$S_{\text{ship}}\left(l_j\right)=\begin{cases}1 & \text{if} l_j \geqslant \text{high water level} \\ -1 & \text{otherwise}\end{cases}$$

Marina Manager and Resident near the Port of Montreal: stable and low water level

$$S_{\text{dock}}\left(l_j\right)=\begin{cases}1 & \text{if} l_j \leqslant \text{low water level} \\ -1 & \text{otherwise}\end{cases}$$

Environmentalists: want seasonally high and low water level

$$S_{\text{env}}\left(l_j, j\right)=\begin{cases}2 & \text{if}\left(l_j \geqslant \text{high water level in spring} \wedge j \in \text{Spring}\right) \vee \left(l_j \leqslant \text{low water level in Winter}\right) \\ -2 & \text{otherwise}\end{cases}$$

Lakefront property owners and recreational boaters: want medium and stable water level

$$S_{\text{rec}}\left(l_j\right)=\begin{cases}1 & \text{if lower bound of medium water level} \leqslant l_j \leqslant \text{higher bound of medium water level} \\ -1 & \text{otherwise}\end{cases}$$

Hydroelectric utilities: want to utilize high water levels during periods of high energy consumption

$$S_{\text{hydro}}\left(l_j, j\right)=\begin{cases}2 & \text{if} l_j \geqslant \text{high water leve} \wedge j \in \text{high consumption months} \\ -2 & \text{otherwise}\end{cases}$$

The penalty function (P) gives a penalty based on whether the water level l_j is outside the normal range

$$P\left(l_j\right)=\begin{cases}k_1 \cdot \left|extreme\_low-level\right| & \text{if } l_j < \text{Extreme low bound} \\ k_2 \cdot \left|normal\_low-level\right| & \text{if Extreme low bound} \leqslant l_j < \text{Normal low bound} \\ 0 & \text{if Normal low bound} \leqslant l_j \leqslant \text{Normal up bound} \\ k_3 \cdot \left|normal\_high-level\right| & \text{if Normal up bound} < l_j \leqslant \text{Extreme up bound} \\ k_4 \cdot \left|extreme\_high-level\right| & \text{if } l_j > \text{Extreme up bound}\end{cases}$$

Where, k_1, k_2, k_3, k_4 are penalty coefficients to adjust the penalty intensity for different degrees of exceeding the range. This penalty function gives the penalty according to whether the water level l_j is out of the normal operation range.

Genetic Algorithm and Scoreboard

Input: I_i , l_j , M_j , p_c , p_m , weights

Output: $F\left(I_i\right)$ (m)

for i=1 to 240 **do**

Test with 20 years of data and setting limits, initial probabilities
Determine weight in peak and non-peak seasons and apply penalties if the result approaches to the limits

Score the optimizing results and backtest
Backwardation to solve the rest of the lakes

end

The maximum theoretical score for each month is as follows:
Winter (December, January, February, need low water level): maximum score of 3.
All other months: maximum score of 1.
Total **maximum score** for a year: 18 and the model get 12 out of 18, while the actual result in 2017 only gets 0.

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Table 4. Optimal Water level (m) of Five lakes in 2017

	Superior	Mich&Huron	Clair	Erie	Ontario
Jan	183.47	176.47	175.19	174.29	74.62
Feb	183.43	176.48	175.22	174.4	74.82
Mar	183.41	176.53	175.29	174.47	75.00
Apr	183.56	176.66	175.47	174.64	75.35
May	183.63	176.80	175.61	174.82	75.43
Jun	183.72	176.93	175.64	174.82	75.62
Jul	183.75	176.99	175.71	174.86	75.80
Aug	183.80	176.91	175.73	174.85	75.69
Sep	183.84	176.81	175.68	174.73	75.33
Oct	183.81	179.79	175.61	174.66	75.08
Nov	183.65	179.73	175.46	174.59	74.86
Dec	183.54	176.67	175.24	174.49	74.77

Table 5. Score

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Model	3	3	-1	-1	1	1	1	-1	1	1	1	3
2017	-1	-1	-3	1	1	1	1	1	1	1	1	-3

Model total score: 12

2017 actual total score: 0

Through the application of a genetic algorithm within the DEAP framework, my methodology presents a novel solution to the complex problem of water level management in Lake Ontario. By intelligently balancing the diverse needs of stakeholders with environmental sustainability, I provide a **robust tool for decision-makers in water resource management**. This approach not only fosters ecological health and societal well-being but also enhances the efficiency and responsiveness of water level adjustments throughout the year.

4.3 Model Backtest

After analyzing the results of the optimization algorithm in comparison to the actual 2017 water level data, the following are the key observations and conclusions about the performance of the algorithm:

(1) In most months, the algorithm tended to recommend slightly higher levels than the actual 2017 water levels. This may point to a more prudent water level management strategy to minimize potential negative impacts and may reflect a more nuanced consideration of the low water demand side.

(2) For the June data, the algorithm received a positive score for month 6, whereas the actual 2017 water levels received a negative score for that month. This suggests that the water levels selected by the model in this month may **better meet specific stakeholder needs,** such as the high-water-level needs of hydroelectric utilities during periods of high energy consumption.

(3) During the high-energy summer months of July through September, water levels in 2017 were lower than those recommended by the algorithm and resulted in negative scores, suggesting that water levels may not have been managed well enough to meet the high water level needs of interested parties in that year. In contrast, the algorithm provided water levels in these months that were closer to the interested parties' expectations, leading to higher scores.

(4) The lowest actual water level scores were recorded in December, suggesting that there may have been a significant deviation from multi-stakeholder needs. In contrast, the algorithm received a higher score for water level selection in that month, showing its **effectiveness** in matching the expected criteria.

Overall, the algorithm demonstrated **good sensitivity and**

adaptability in trying to balance the diverse water level demands of different months throughout the year.

5 Model II: Network Flow Model

5.1 Net Basin Supply

In order to utilize the Network Flow Model, I combined the water balance equation and the Net Basin Supply (NBS) to better visualize each lake's. The water balance equation provides a comprehensive overview of the:

$$Q_{\text{out}} = Q_{\text{in}} + P - E - D \quad (3)$$

Where Q_{out} is the outflow. If it exceeds the threshold, it will cause flooding. Q_{in} is the inflow, including groundwater and the water from upstream. P is the total precipitation on the lake surface, measured in millimeters (mm). E is the evaporation from the lake surface (mm). D is the net diversion. I will use this equation to make decisions about opening or closing the gates of the dams.

The NBS is another important element that help us to better understand the source and sink flow. For each lake, NBS is calculated using the formula: Component NBS calculation – commonly used

$$NBS = P + R - E + G$$

Where R represents the surface runoff entering the lake, aggregated from tributary inflows, expressed in m^3/s. G denotes the net groundwater flow into or out of the lake, also in m^3/s. Given the challenges in directly measuring G, this study uses regional estimates adjusted for the lake area.

For precipitation and evaporation, the conversion from depth (mm or inches) to volumetric flow rate (m^3/s) involves the lake surface area (A), given in square kilometers (km^2), and the duration of the period (t, in seconds):

$$\text{Volume} = \frac{\text{Depth} \times A \times 10^6}{t}$$

5.2 Model Result

The network flow model encapsulates the flow dynamics within the interconnected river systems. This model serves a dual purpose: first, it enables the maintenance of a hydrological equilibrium across the network; second, it ascertains that the water flow through **each river conduit is optimized**. The essence of this optimization is to align with the energy sector's imperative for maximizing flow rates, thus catering to the hydroelectric power generation demand. My model is designed to not only uphold this equilibrium but also to ensure that the flow requirements are met in a manner that does not compromise the system's integrity, thereby maximizing the utility of the natural resource while sustaining its ecological balance.

Table 6. River Flow (m^3/sec) in 2017

	St. Mary	St.Clari	Detroit	Niagara	Ottawa	St.Lawrence
Jan	2256.3	5649.2	6088.2	6320.4	1967.2	6229.7
Feb	2431.0	5889.9	6158.9	6540.3	2187.5	6711.1
Mar	2267.3	5907.4	6297.9	6510.2	3240.5	7447.3
Apr	1998.4	5977.7	6283.5	6880.6	5650.4	7787.1
May	1998.7	6028.3	6164.6	7320.5	6337.9	8580.0
Jun	2399.6	6221.2	6266.5	7200.3	3253.2	10222.4
Jul	3157.3	6287.3	6269.4	7170.2	2622.8	10392.6
Aug	2681.6	6286.3	6342.9	6940.9	1540.9	9599.4
Sep	2690.4	6240.9	6365.6	6580.6	1600.4	8636.6
Oct	2961.9	6130.6	6592.1	6610.7	1353.8	8468.7
Nov	2248.9	6232.5	6454.3	6820.6	2645.4	8353.5
Dec	2234.7	6054.4	6351.5	6800.1	2273.5	6237.8

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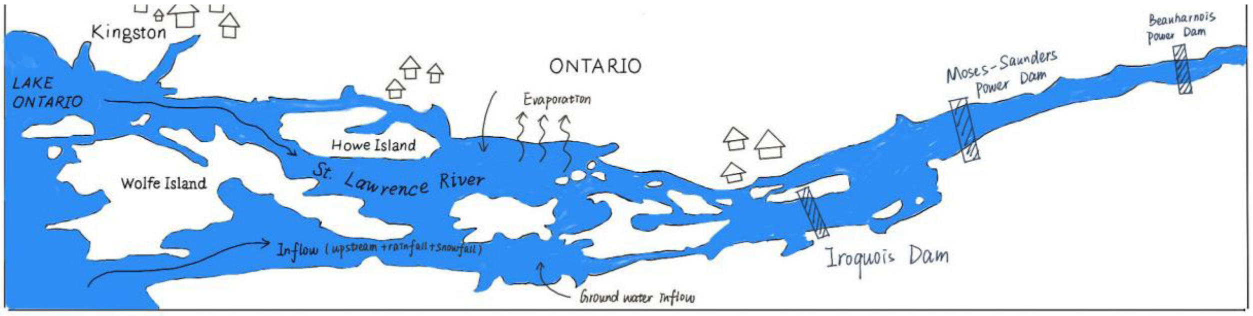


Figure 4. Network Flow Graph

Logics of controlling dams:

The proposed damoperation logic follows a straightforward feedback principle: if the current water level plus the net inflow exceeds the calculated optimal water level, the dam gates are closed to retain excess water upstream. Conversely, if the sum of the actual water level and net inflow falls below the optimal target, the gates are opened to release water until the desired level is restored. Through this responsive regulation, river stages can be consistently maintained at a stable and operationally optimal elevation.

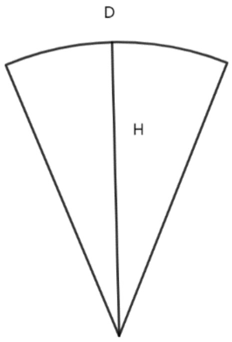
6 Model III: Multi-target Optimize Model with Ford-Fulkerson Algorithm

6.1 Modeling Process

6.1.1 Thoughts

At the outset of the analysis, the provided dataset underwent meticulous preprocessing to clearly segregate lake waterlevel records from river discharge data. This separation is essential to distinguish between the relatively static water levels of the lakes—hereafter termed "waterlevel data"—and the dynamically varying water flows in the rivers, referred to as "waterflow data."

A foundational step in the study involves elucidating the relationship between water levels and water flows across the lake system. Fundamentally, a lake's water level reflects its storage volume. To quantify this relationship, the complex bathymetry of each lake is abstracted into a schematic crosssection, approximating the lake geometry as a **conical shape**. This simplification facilitates a tractable yet physically meaningful estimation of volume. The conceptual basis of this geometric abstraction is outlined below:



The slope of the cone is constant and the diameter is proportional to the depth:

$$D = kH$$

$$V = \frac{\pi}{3} H \left(\frac{D}{2}\right)^2$$

The rate of change of height H is related to the rate of change of V

$$\frac{dV}{dt} = \frac{\pi}{4} k^2 H^2 \frac{dH}{dt}$$

In the course of my investigation into the hydrological dynamics of the Great Lakes system, I introduced a suite of functions, designated v_1 to v_5, each crafted to calculate the volume of water corresponding to a given height h within different segments or constituents of a water system.

Each function hinges on a **distinct ratio**, denoted by k, that is, the height to base diameter of the assumed conical representation of the water bodies. The volumetric formula employed across these functions is articulated as:

$$V = k^2 \times \frac{\pi}{12} \times h^3$$

After that, a suite of functions was designed to quantify and analyze the **variations in water flow between consecutive segments** within a water system. The dv(w) function is crafted to calculate the differential flow rates across a series of channels or segments, where w represents a list or array of water flow rates for six distinct segments, denoted as w1 to w6. The output of this function is a list of flow differentials, with the first element (-w1) representing the outflow from the first segment. Subsequent elements reflect the net flow between adjacent segments (e.g., w1-w2, w2-w3, and so on), with the final element (w4-w5-w6) capturing the combined outflow from the fifth segment into the sixth and beyond. This function is essential for understanding the distribution of water throughout the system and identifying potential bottlenecks or surplus areas.

Following that, the **maxflow(w) function** is utilized to assess the total flow within the system. By summing the flow rates across all segments, it outputs a singular value representing the system's total flow. This assessment is crucial for understanding the overall capacity or throughput of the water flow system.

Lastly, the **miniheight(h, dh) function** calculates a volume based on the heights h and height changes dh across five segments, factoring in specific ratios for each segment. The function outputs the sum of these calculations for the system's total volume.

The **objective function**, denoted as $L(x)$, is expressed as the sum of certain elements from the 7th to the end of the input vector x, augmented by the first element of x multiplied by a factor derived from constants. Specifically, the function is articulated as:

$$L(x)=\sum_{i=7}^{n} x_i + \frac{4x_0}{\pi \cdot (244)^2}$$

This formulation represents a cost that necessitates minimization to achieve optimal water flow or distribution within the system.

The optimization is subject to a series of inequality constraints that ensure the feasibility of solutions from an operational standpoint. Each constraint, encoded as a lambda function, enforces conditions on the variables x to satisfy certain balance or flow requirements between different sections of the system. The **constraints** are as follows:

1 $x_0 + x_5 \geqslant 0.$

2 $x_1 - x_5 + x_6 \geqslant 0$

3 $x_2 - x_6 + x_7 \geqslant 0$

4 $x_3 - x_7 + x_8 \geqslant 0$

5 $x_4 - x_8 + x_9 + x_10 \geqslant 0$

x_0： the water flow of St. Mary's River.

x_1： the water flow of St. Clair River

x_2： the water flow of Detroit River

x_3： the water flow of Niagara River

x_4： the water flow of Ottawa River.

x_5： the water flow of St. Lawrence River

x_6： the water level of Lake Superior

x_7： the water level of Lake Michi&Huron

x_8： the water level of Lake St. Clair.

x_9： the water level of Lake Erie

x_{10} : the water level of Lake Ontario

These constraints are instrumental in maintaining the net flow or balance between various system sections, reflecting the inherent limitations on water redistribution or management and preventing the problem of river water flowing backward.

The optimization problem incorporates **specific bounds** for each variable x, derived from preliminary estimates or measurements adjusted by a margin. These bounds are critical in ensuring the optimizer's exploration remains within realistic and feasible solution spaces. The bounds are established as:

$$min(a_i, b_i) \leqslant x_i \leqslant max(a_i, b_i) \text{ for } i = 0,1,\ldots,n$$

Where a_i and b_i represent arrays of lower and upper bounds, respectively, providing a controlled range for each decision variable.

6.2 Optimization Execution

The optimization process is executed via the minimize function, aiming to identify values of x that **minimize** $L(x)$ while adhering to the defined constraints and operating within the prescribed bounds. The initial guess for the optimization is set as array a, providing a starting point to guide the iterative search.

In this comprehensive analysis of water flow and distribution optimization across a sequential timeframe, an iterative methodology is adopted to solve a series of constrained optimization problems. This approach is crucial for accommodating temporal variations in system parameters and operational conditions, thereby ensuring that the optimization remains dynamically relevant and aligned with real-world operational dynamics.

To support multicriteria decisionmaking within this framework, the Analytic Hierarchy Process (AHP) is employed. AHP operates on a pairwise comparison matrix, denot-

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ed A, in which each entry quantifies the relative importance or preference between two criteria (or alternatives). The process begins by evaluating the rank of A and computing its eigenvalues and eigenvectors. The eigenvector corresponding to the largest eigenvalue is then normalized to yield a set of priority weights Q for the criteria or alternatives. These weights provide a quantitative foundation for systematic and consistent decisionmaking throughout the optimization procedure.

The entire model is illustrated as such:

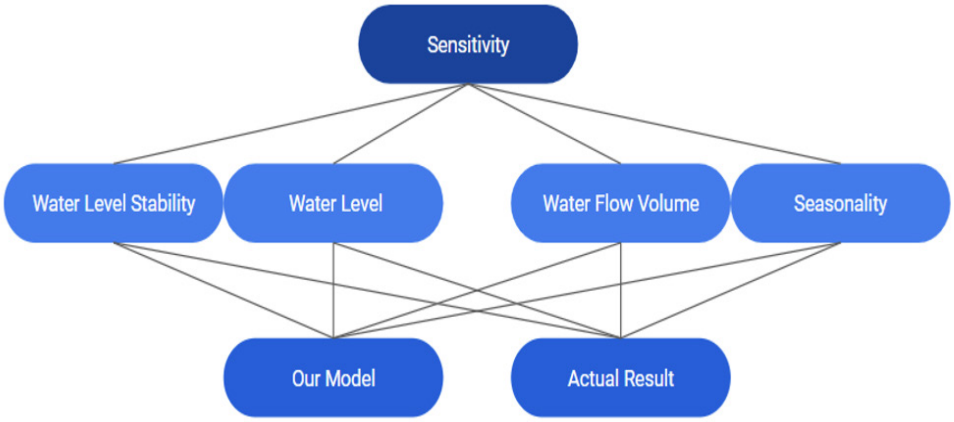

Figure 5. Analytic Hierarchy Process

Based on the theory of AHP, I adopted decision matrix and the 1 represents the two adjacent elements are equally important, 3 represents that the former is slightly more important than the latter and 5 means that the former element is much more important than the latter.

$$\begin{bmatrix} 1 & \frac{1}{5} & \frac{1}{3} & 1 \\ 5 & 1 & 3 & 5 \\ 3 & \frac{1}{3} & 1 & 3 \\ 1 & \frac{1}{5} & \frac{1}{3} & 1 \end{bmatrix}$$

Next, hierarchical single sorting is conducted. This process involves pairwise evaluation of all elements within a given layer with respect to a criterion from the preceding layer, resulting in a prioritized ranking of their relative importance. Specifically, the judgment matrix A is used as the computational basis, ensuring that the resulting eigenvector satisfies the eigenvalue equation $AW = \lambda W$. Here, $\lambda_{\max}$ denotes the largest eigenvalue of A, and the corresponding regularized eigenvector W provides the weight values for each element—this constitutes the hierarchical single sorting. Thus, the judgment matrix serves to calculate the weight coefficients of each factor relative to the target criterion, establishing a quantified basis for priority assessment.

The procedure of calculating the weight vector (W) and the maximum feature (λmax) (square root method or sum method) is shown in the following table:

Table 7: Procedure of calculating

	Calculation Steps	Equations
a	Product by row elements, then to the power of 1/n	$\bar{Wi}\sqrt{\prod_{i=1}^{n} a_{ij}}, i, j = 1, 2, \ldots, n$
b	Normalize Wi (so that the sum of the elements in the vector is equal to 1) that is, the ranking weight vector, denoted as W. (The elements of W are the ranking weights of the factors at the same level with respect to the relative importance of a factor at the previous level, then W = (W1, W2,.....) W = (W1, W2,)^t is the required eigenvector, which is also the result of the hierarchical single sorting of the judgment matrix.	$Wi = \bar{W}i / \sum_{i=1}^{n} Wi$
c	Determine the largest characteristic root of the matrix	$\lambda\max = 1/n \times \sum_{i=1}^{n} (AWi/Wi)$

To ensure the reliability of these pairwise comparisons, I calculate a Consistency Index (CI), which gauges the degree of deviation from perfect consistency in judgments. This index is subsequently compared against a Random Index (RI), an average consistency metric derived from randomly generated matrices, to compute the **Consistency Ratio**.

Let the nth order judgment matrix be B, then its largest characteristic root can be found by :

$$BW = \lambda W$$

where W is the eigenvector of B. In hierarchical analysis, I use the following Consistency Index (CI) to test the consistency index of the judgment.

$$C.I = \frac{\lambda max - n}{n-1}$$

C.I.=0 indicates that the judgment matrices are in perfect agreement, and the larger C.I. is, the more serious the degree of inconsistency of the judgment matrices is.

Solve for the CR value based on the CI and RI values and determine whether the consistency passes or fails:

$$C.R. = \frac{C.I.}{R.I.}$$

When C.R.<0.1, it indicates that the degree of consistency of judgment matrix A is considered to be within the tolerance range, and then the eigenvectors of A can be used to carry out the calculation of the weight vector; if C.R.⩾0.1, then consideration should be given to correcting the judgment matrix A.

Multi-Target optimize Model

Input: x_i, k, h, weights

Output: Monthly max and min flows

for i=1 to 108 **do**

    Treat each lake as a cone to calculate volumes
    setting constraint functions with boundaries (a,b)
    Minimized L(x) to get the water flows with AHP
    Considering the weather conditions, such as snowpack, ice jams, to change α
    Using FF algorithm to determine the optimal water level by considering flows

end

6.3 Result Interpretation

The **RI matrix** is generated by Satty for 1000 times: RI=[0, 0, 0.58, 0.90, 1.12, 1.24, 1.32, 1.41, 1.45, 1.49, 1.51]. In case, C.R. are only slightly larger than 0.01, so I was allowed to proceed to calculate the weights.

Weights are [0.09546368 0.55958606 0.24948658 0.09546368], and with individual weights W1 to W4, I can get the **final score** for model is 0.80240 and the score for the original data in 2017 is 0.19759, suggesting that the current model performance is largely greater than the existing natural water condition.

7 Model Verification

7.1 Sensitivity Analysis

In this study, an innovative approach was developed to analyze and optimize water resource management within the Great Lakes system, with particular emphasis on the effects of precipitation (rain and snow) and ice formation on hydrological dynamics. By integrating mathematical modeling with optimization techniques, the research aims to investigate how varying environmental conditions influence water flow and levels, and to propose actionable management strategies in response to these changes. Through the formulation of constraint sets, the impacts of increased precipitation and iceinduced flow reduction on water discharge rates were systematically simulated, enabling subsequent adjustments in water allocation to achieve an optimized operational state. These constraints were designed to mirror realworld physical limits and managerial requirements, thereby ensuring that the optimization outcomes remain both practical and implementable under actual operating conditions.

7.1.1 Water Resource Optimization under Precipitation

Rain and snow are treated collectively as increased precipitation within the model framework. The hydrologic

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influence of precipitation depends primarily on the surface area of each lake and the precipitation intensity (density). Multiplying these two factors yields the additional water volume to be incorporated into the model, thereby enhancing its accuracy and completeness under varied precipitation scenarios.

$$W = \rho\pi\left(\frac{D}{2}\right)^2$$

where ρ is the density of precipitation. Naturally, the water flow constraints became:

$$\frac{d}{dt}Vs = -w1 + ws$$

$$\frac{d}{dt}V_{wmn} = w1 - w2 + WMH$$

$$\frac{d}{dt}Vc = w2 - w3 + wc$$

$$\frac{d}{dt}Ve = w3 - w4 + we$$

$$\frac{d}{dt}Vo = w4 - w5 - w6 + w0$$

Where w_{MH}: the water level of Lake Michigan and Lake Huron; w_e: the water level of Lake Erie; w_s: the water level of Lake Superior

The water level for five lakes became slightly higher, which not only shows that my model is responsive to precipitation circumstances, but also shows that the model is stable under severe situations.

When rainfall density= 0.1:

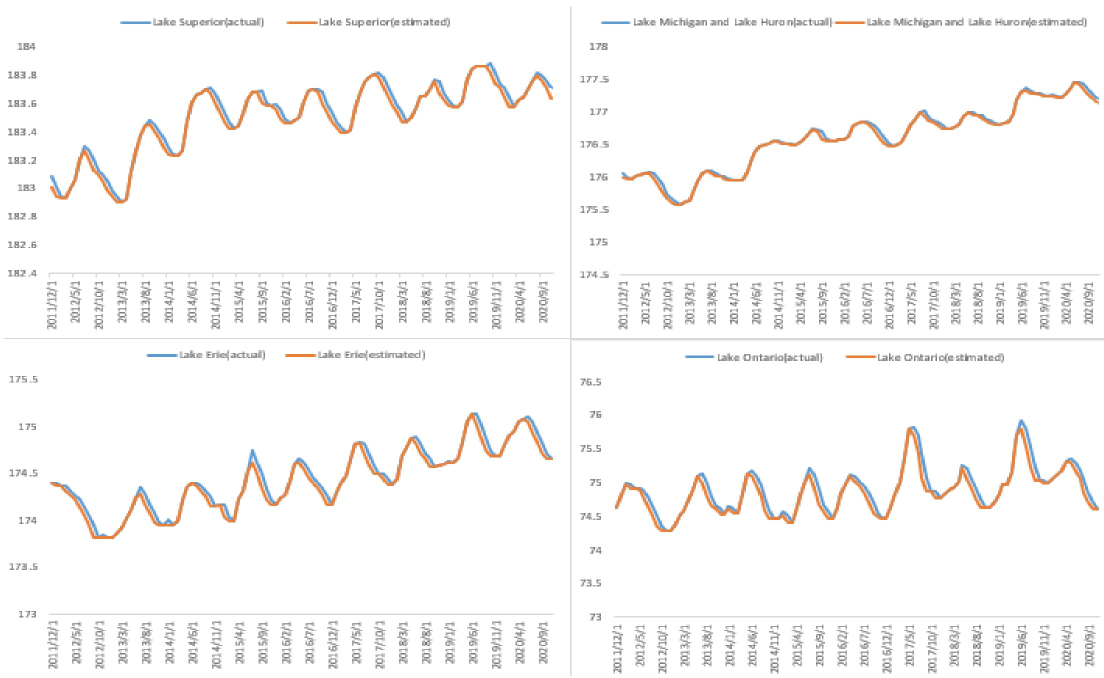

Figure 6. Rainfall condition

In accordance with the observed rainfall frequency in the Great Lakes region, precipitation intensity was estimated and incorporated into the model through sensitivity analysis coefficients of [0.01, 0.05, 0.1]. The results demonstrate a high degree of consistency with collected empirical data, confirming that the model effectively captures the principal drivers of waterlevel fluctuations, particularly during precipitationdominant periods. As illustrated in the accompanying figures, the predicted waterlevel trajectories align closely with observed trends, underscoring the model's sensitivity and predictive accuracy.

Nonetheless, certain deviations are noted. For example, in Lake Erie, the predicted water level for August 2015 was 175.8, whereas the observed value was 174.4, reflecting a modest overestimation of peak conditions. Over the period from November 2011 to November 2020, the overall fit across all lakes remains strong; however, a recurrent lag is evident in observed water levels relative to predictions during the springtosummer transition. A case in point is Lake Superior, where the predicted level on July 1, 2016 was 183.71, while the observed level did not reach 184 until September 1.

Importantly, under varying precipitationintensity scenarios, the model outputs show no substantial divergence in predicted water levels across the Great Lakes system. Historical records further indicate an absence of extreme weather episodes in the region in recent years, lending additional support to the model's structural plausibility and overall reasonableness.

7.1.2 Water Resource Optimization under Ice Condition

To simulate the hydrological influence of ice formation, distinct ice-impediment coefficients (e.g., 0.1, 0.05, 0.01) were introduced into the model. These coefficients quantitatively represent the flowretarding effect of ice cover on lake and river systems. Incorporated as modifiers within the optimization constraints, they explicitly account for iceinduced reductions in water discharge, thereby allowing a systematic evaluation of the adaptability and robustness of waterresource management strategies across a range of iceseverity scenarios.

For illustration, when the snowimpediment coefficient is set to 0.1, the model simulates the corresponding flow conditions as follows:

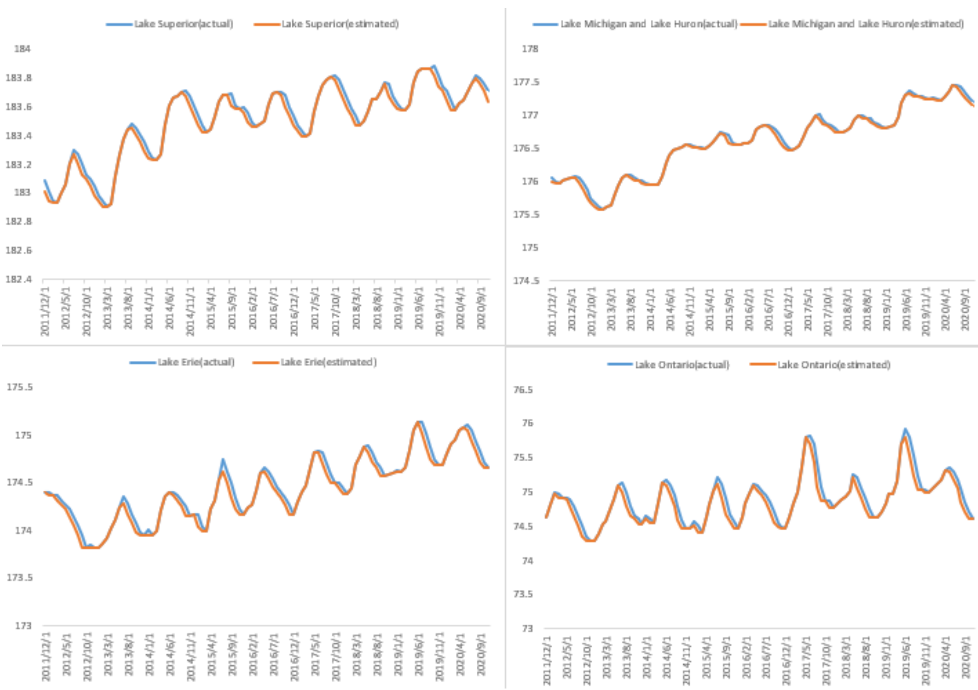

Figure 7. Snowing Condition

The analytical results indicate that the model maintains strong consistency with observed data across varying icecover scenarios, with predicted waterlevel curves closely tracking both the fluctuation patterns and trends of the actual measurements. This demonstrates the model's capability to capture the principal dynamics driving waterlevel variations. For example, the predicted level for Lake Ontario on June 1, 2019 was 74.1, compared to an observed value of 75.99, while for Lake Superior on April 1, 2015 the prediction was 183.3 against an observed 183.5. These comparisons reveal modest deviations in peaklevel estimates, which may partly arise under extreme or transient hydrological conditions.

Furthermore, the analysis suggests that variations in iceflow resistance coefficients exert only a minor influence on simulated water levels, implying that ice formation does not directly alter Great Lakes water levels in a substantial manner. Although precipitation in the form of rain and snow generally has limited impact on lake levels under typical conditions, extreme weather events and seasonal transitions remain significant drivers of waterlevel fluctuations. Notably, even when averaging across diverse precipitation scenarios, rain and snow contribute minimally to mean waterlevel changes; however, episodic extremes and seasonal hydrological shifts continue to represent important factors affecting the water balance of the Great Lakes system.

8. Conclusion

In summary, my model demonstrates high adaptability and accuracy in simulating water level changes in the Great

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Lakes, despite potential slight predictive discrepancies under extreme weather or specific seasonal conditions.

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