S-AT-KiBaM-Age: A Dual-Time-Scale Integrated Model for Smartphone Battery

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Abstract Predicting smartphone battery life remains difficult because hardware load, environmental conditions, and battery aging interact in complex ways. Most current methods rely on curve- fitting or black-box models that lack physical grounding and fail to capture real-world usage patterns. We propose the S-AT-KiBaM-Age model, a continuous-time framework that couples macro-scale aging with micro-scale discharge dynamics through thermo-electro-kinetic interactions. The model embeds the Kinetic Battery Model for charge transfer, applies the Arrhenius equation for temperature-dependent resistance, and tracks State of Health degradation through an empirical aging formula. By quantifying energy dissipation pathways—Joule heating, trapped charge, and voltage collapse—it identifies what drives rapid battery drain. Validation on the AndroWatts dataset shows an RMSE of 0.183 hours (2.86% relative error) across 24 usage-environment-aging scenarios. Sobol sensitivity analysis reveals that screen brightness (first-order index 0.55), ambient temperature, and battery aging dominate battery life, while background app management improves runtime by less than 1%. Users can extend battery life by 23-35% by keeping screen brightness at 30-50%, avoiding temperatures below 0°C or above 40°C, and using Wi-Fi instead of cellular in weak signal areas. The model generalizes to other portable devices via parameter scaling and enables physics-aware Model Predictive Control for OS-level power management. This work offers a physically grounded tool to reduce battery anxiety and improve energy efficiency.

Keywords Smartphone Battery; Continuous-Time Model; Dual-Time Scale; Thermo-Electro Coupling; SOC Prediction; Battery Aging; Power Consumption Optimization

1 Introduction

Smartphones have evolved from luxury items to indispensable daily tools, serving as the morning alarm, navigation assistant, work communicator, and entertainment hub. Yet this deep dependency has given rise to a widespread anxiety: when the battery indicator turns red, many people instinctively search for a charger, as if losing a sense of security. This “battery anxiety” has become a distinctive psychological phenomenon of the digital age. Battery life is more complex than just capacity. It stems from the nonlinear coupling of multiple factors: lithium-ion batteries show temperature-dependent internal resistance and nonlinear discharge curves; OS scheduling and network handoffs draw continuous power; user behavior varies wildly between gaming and standby. What makes this harder is how these factors interact—cold weather not only cuts battery efficiency but also triggers protection. mechanisms that shorten life further.

This paper built a discharge model from lithium-ion electrochemistry that couples temperature, load, and aging, calibrating parameters against experimental data. Second, we designed a scenario recognition and battery life prediction algorithm using historical behavior data, providing personalized time-to-empty estimates for different usage patterns. Third, through sensitivity analysis, we identified key factors affecting battery life and developed practical power-saving recommendations. Our model balances physical accuracy with computational efficiency, providing a foundation for practical battery management apps.

2 Model Preparations

Assumption 1: Electrochemical Invariancez:The battery's core electrochemical structure remains unchanged throughout its life.

Justification: The core chemical mechanisms and material composition persist despite performance degradation, ensuring the validity of kinetic equations across all cycles.[1]

Assumption 2: Constant Kinetic Parameters per Cycle KiBaM parameters including capacity ratio and charge transfer rate are treated as constants during a single discharge event. **Justification**: Discharge dynamics occur on a timescale orders of magnitude faster than aging processes, allowing parameters to update only between cycles.

Assumption 3: Invariant Physical Properties We define physical attributes like battery mass and specific heat capacity as time-invariant constants.

Justification: These intrinsic material properties and physical dimensions do not fluctuate with state of charge or temperature.

Assumption 4: Stable Ambient Temperature The ambient temperature is considered constant during any single discharge session.

Justification: The thermal inertia of the battery mass prevents rapid core temperature changes resulting from short-term ambient fluctuations.

Assumption 5: Linear Power Superposition Total smartphone power consumption is modeled as the linear sum of individual component power draws.

Justification: Modern power management units regulate component voltage rails independently which allows for additive power calculation

Assumption 6:Negligible Self-Discharge We omit self-discharge currents from our charge conservation equations.

Justification: The magnitude of active operational current is significantly larger than natural leakage rates over the modeled timescale.

Assumption 7:Constant Conversion Efficiency We fix the PMIC conversion efficiency as a constant value, independent of instantaneous load or temperature.

Justification: Modern converters maintain high stability across operating ranges and a fixed efficiency enables closed-form mathematical solutions.[2]

Assumption 8: Markovian User Behavior We simulate user activity as a Markov process with stationary transition probabilities.

Justification: Empirical studies confirm that smartphone usage patterns exhibit statistical stability and predictable distributions over daily cycles.[3]

3 Model Design

3.1 Preprocessing

To address the continuous-time modeling requirements of smartphone battery consumption, we propose the **S-AT-KiBaM-Age model** (Stochastic-User Adaptive Thermal-Kinetic Battery Model with Aging Evolution).

The model employs a **dual-timescale framework**:

Macroscopic scale (N-scale): Captures the battery aging evolution process—encompassing State of Health (SOH) degradation, capacity fade, and internal resistance increase—as a function of cycle number N;

Microscopic scale (t-scale): Characterizes the electrochemical kinetics, thermodynamic responses, and circuit-level electrical behavior during a single discharge event.

The model architecture consists of three interconnected sub-models:

Macroscopic-scale Aging Model: Updates the battery's physical parameters prior to each discharge cycle;

Microscopic-scale Stochastic Load Model: Converts uncertain user behaviors into a continuous-time power function $P_{load}(t)$;

Core Electro-Thermal Dynamic Cxoupling System: The central component that solves for battery states at each microscopic time step dt.

3.2 Prediction of battery usage time under different state

To address the challenge of nonlinear power consumption prediction for smartphone batteries under complex load conditions, this study establishes the S-AT-KiBaM-Age model(Stochastic-User Adaptive Thermal-Kinetic Battery Model with Aging Evolution) . The framework comprises three progressively sophisticated sub-models structured in a hierarchical cascade: the first sub-model captures the fundamental intrinsic electrochemical discharge mechanisms; the second incorporates user-interactive consumption dynamics; and the third integrates coupled thermal-aging effects, thereby escalating from basic physical principles to a comprehensive, reality-grounded predictive system.

3.2.1 Model I: Macro-Scale Aging Model

The State of Health (SOH) is jointly influenced by Cycle Aging and Calendar Aging. We establish the following empirical degradation model:

$$SOH(N) = 1 - \alpha_{cyc} \times N^{\beta} - \alpha_{cal} \times t_{shelf}$$

α_{cal}: Calendar aging coefficient

t_{shelf}: Battery shelf time (idle time)

The decrease in SOH modifies the parameters of the micro-scale model through the following two mechanisms:

1.Capacity Fade Mechanism:

$$Q_{\max}(N) = Q_{design} \times SOH(N)$$

2. Internal Resistance Rise Mechanism:

$$R_{aging}(N) = 1 + \kappa \times (1 - SOH(N))$$

This equation describes the irreversible energy loss occurring in the battery as the number of cycles increases.[4]

3.3 Model II: Micro-Scale Stochastic Load Model

Due to the diversity of user interactions with smartphones, each operation exhibits distinct power consumption characteristics. However, these behaviors fundamentally rely on a common set of hardware components—namely the CPU, screen brightness, and network modules. Therefore, we abstract complex user behaviors into discrete load categories, and quantify energy consump-

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tion by aggregating the active status of these individual loads.

3.3.1 User State Machine and Power Synthesis

The total power is superposed by the base power consumption and the dynamic power consumption of each component:

$$P_{load}(t)=P_{base}+P_{screen}(t)+P_{cpu}(t)+P_{rf}(t)$$

3.3.2 Component Physical Power Consumption Equations

To achieve the endogenization of loads, we construct the power consumption equations for each component based on physical principles:

Screen Power Consumption: The screen brightness B(t) ∈ [0,100] is modeled as a stochastic process.

$$P_{screen}(t)=A_{disp}\times\left[C_0+C_1\times\left(\frac{B(t)}{100}\right)^{\gamma}\right]$$

where γ≈2.2 is the gamma correction coefficient, reflecting the nonlinear relationship between human visual perception and power consumption.[5]

Processor Power Consumption: Based on the CMOS dynamic power formula P∝CV2f 。Let the CPU utilization be u(t) , and the frequency,and $f(t) \propto u(t)$：

$$Pcpu(t)=Pleak+Cdyn \times u(t)\times f(t)2$$

RF Power Consumption: When the signal strength $S_{net}(t) \in [0,1]$decreases, the transmit power increases exponentially to maintain the connection:

$$P_{rf}(t)=P_{idle}+P_{tx}\times e^{\lambda(1-S_{net}(t))}$$

With this model, user interactions with the smartphone can be systematically incorporated into the analytical framework.

3.4 Coupled Thermo-Electro-Kinetic System

Given the influence of temperature on internal resistance, variations in thermal conditions will alter both the battery's operating duration and terminal voltage. Should the voltage drop below a critical threshold, the system becomes inoperable, triggering voltage collapse.

3.4.1 Dynamic Parameter Update

At each time instant t, calculate the instantaneous internal resistance based on the current temperature $T(t)$ and the aging factor $Raging(N)$:

$$R_{int}(N,T)=R_{ref}\times R_{aging}(N)\times\exp\left[\frac{E_a}{R_g}\left(\frac{1}{T(t)}-\frac{1}{T_{ref}}\right)\right]$$

3.4.2 Nonlinear Circuit Solver

Given the load power $P_{load}(t)$, solve for the circuit current $I(t)$.

1. Energy Conservation Equation (considering the conversion efficiency η of the PMIC):

$$V(t)\times I(t)\times\eta=P_{load}(t)$$

2. Thevenin Voltage Equation:

$$V(t)=E_{ocv}(SOC)-I(t)\times R_{int}(N,T)$$

3. By solving the above two equations simultaneously, we obtain:

$$(\eta R_{int})\times I(t)^2-(\eta E_{ocv})\times I(t)+P_{load}(t)=0$$

4. Current Solution and Voltage Collapse Criterion:

$$I(t)=\frac{\eta E_{ocv}-\sqrt{(\eta E_{ocv})^2-4\eta R_{int}P_{load}(t)}}{2\eta R_{int}}$$

When Δ < 0，Voltage collapse occurs because no real solution exists for the current.. This implies that the internal resistance is excessively high or the load is too heavy, such that the battery cannot deliver the required power, and the system will be forced to shut down.。

3.4.3 State Evolution via Differential Dynamics

The evolution of the system state vector X=$[y_1, y_2, T]T$ is governed by the following system of ODEs:

Equation Set A: KiBaM

$$\begin{cases}\dfrac{dy_1}{dt}=-I(t)+k\times(h_2-h_1)\times Q_{max}(N)\\ \dfrac{dy_2}{dt}=-k\times(h_2-h_1)\times Q_{max}(N)\end{cases}$$

Height Definition:

$$h_1=\frac{y_1}{c\times Q_{max}(N)},\quad h_2=\frac{y_2}{(1-c)\times Q_{max}(N)}$$

Aging Effect: As the cycle number N increases, the maximum capacity $Q_{max}(N)$ diminishes. Consequently, for the same amount of remaining charge y , the potential head h increases, creating a "virtual high voltage" phenomenon; however, the charge y is depleted at a significantly faster rate.

Equation Set B: Thermodynamic Energy Balance

$$mC_p\frac{dT}{dt}=\Phi_{gen}(t)-\Phi_{diss}(t)$$

Expanding the heat generation(Φgen)and dissipation(Φ-gen) terms:

$$mC_p\frac{dT}{dt}=$$

$$\underbrace{\left[I(t)^2R_{int}(N,T)+P_{load}(t)\left(\frac{1}{\eta}-1\right)\right]}_{\text{Heat Generation (Joule + Conversion)}}\underbrace{h_{cool}A\left(T(t)-T_{amb}\right)}_{\text{Convective Cooling}}$$

1 Note: The term $P_{load}(t)(\eta-1)$ accounts for the heat generated by the PMIC inefficiency, a critical factor in smartphones.

3.5 Termination Conditions of the Model

3.5.1 Initial Conditions

$$\begin{cases}y_1(0)=c\times Q_{max}(N)\\ y_2(0)=(1-c)\times Q_{max}(N)\\ T(0)=T_{amb}\end{cases}$$

3.5.2 Termination Criteria

The discharge termination time tend $tend$ is defined as the first occurrence of any of the following events:

Depletion:SOC(t) ⩽ 0%

Cut-off:V(t) ⩽ V_{cutof} (Here we set 2.7v)[6]

Thermal Shutdown: $T(t) \geq T_{safe}$ (Here we set 60°C)[7]

Under these circumstances, the battery is considered depleted. We utilize the aforementioned model to calculate the time elapsed from the initial conditions to the termination criteria; this duration is defined as the usage time, serving as our prediction for the battery life.

3.6 The Model Results

Based on the model described above, we derive the following equations:

$$I(t) = \frac{\eta E_{ocv} - \sqrt{\left(\eta E_{ocv}\right)^2 - 4\eta R_{int} P_{load}\ (t)}}{2\eta R_{int}}$$

We utilized extensive real-world data from the AndroWatts dataset[8] and assumed a battery capacity of 4422 mAh, equivalent to that of an iPhone 15 Pro Max battery[9]. Employing the pandas library in Python for iterative computation and solution, we obtained the following parameters after 10 iterations:

	c0	c1	c2	c3	c4	c5	c	k	κ	α_cyc	C0	C1	P_leak	C_dyn	P_idle	P_tx	λ	η	h_cool
Run 1	3.0919	6.442	-37.0306	89.38	-92.7202	36.334	0.5483	0.06076	0.32434	0.00681	223708.49	272761.91	354327.86	5174.51	145526.35	58526.46	0.2993	0.9287	0.01985
Run 2	3.1636	6.6903	-36.3368	87.8237	-94.1199	36.0558	0.5457	0.05939	0.32127	0.00697	229205.74	271341.29	355568.97	5133.96	141484.09	56983.65	0.3025	0.9044	0.01967
Run 3	3.1364	6.6544	-36.561	87.448	-92.3914	36.291	0.558	0.05892	0.31752	0.00699	226279.35	279835.08	347475.74	5112.74	142880.3	56765.24	0.3025	0.9226	0.02031
Run 4	3.1198	6.4921	-36.67	90.5956	-95.6735	36.2271	0.5476	0.06087	0.3141	0.00693	228044.11	278491.32	344564.42	5157.94	141763.35	57560.19	0.3013	0.9121	0.02028
Run 5	3.0647	6.4842	-36.8018	90.6552	-93.2332	35.8022	0.5459	0.06061	0.31727	0.00699	231669.64	276560.21	346214.79	5283.6	141672.13	58705	0.296	0.9275	0.01978
Run 6	3.0647	6.4846	-37.2852	90.0952	-94.7477	36.2658	0.5517	0.06066	0.31745	0.00691	225688.14	279191.4	349003.2	5154.95	140261.77	56989.4	0.3049	0.9102	0.0201
Run 7	3.0525	6.5162	-36.4252	88.3012	-93.4317	35.0734	0.5428	0.06076	0.32263	0.00692	227151.91	278442.15	354475.9	5195.55	143519.16	57981.67	0.2988	0.9292	0.02023
Run 8	3.153	6.5739	-36.8872	87.5642	-94.2133	35.2272	0.5574	0.05909	0.32145	0.00689	230288.33	271621.47	355073.92	5233.8	142911.19	58188.12	0.2972	0.918	0.02002
Run 9	3.1201	6.5496	-37.002	89.6531	-94.3132	35.0115	0.5414	0.05977	0.32464	0.00678	225501.91	279424.6	343121.03	5163.34	140344.77	56979.21	0.2954	0.9383	0.02
Run 10	3.1334	6.5128	-36.2001	88.7839	-92.9559	35.4122	0.5614	0.05919	0.31933	0.00681	224122.4	275520.58	350174.16	5289.6	141636.89	58111.06	0.302	0.9089	0.01977
Average	3.11001	6.54001	-36.72	89.03001	-93.78	35.77002	0.55002	0.060002	0.32	0.0069	227166	276319	350000	5189.999	142200	57679	0.29999	0.91999	0.020001

Figure 3. Statistics of 10 iterations

4 Time-to-Empty predictions

4.1 Model Accuracy

4.1.1 quantify uncertainty

To assess predictive fidelity, we mapped the model's performance onto a 3D response surface across multiple dimensions. As illustrated in Figure 4:

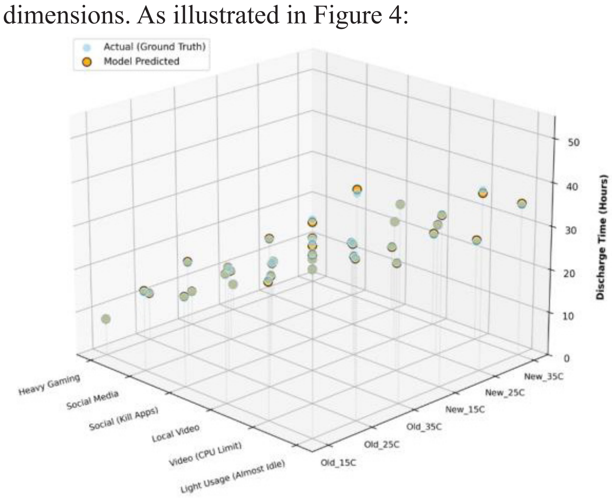

Figure 4. Validation of the S-AT-KiBaM-Age Model: Predicted vs. Actual Values

Figure 4 illustrates a three-dimensional comparative analysis, benchmarking our model- derived predictions against empirical ground-truth data from the AndroWatts dataset. The high degree of convergence between the predicted orange markers and blue empirical points across diverse thermal ranges, battery health profiles, and usage scenarios visually substantiates the model's accuracy. This tight alignment across the multi-dimensional parameter space confirms the exceptional fidelity of the S-AT-KiBaM-Age framework and validates its effectiveness in capturing complex battery discharge kinetics.

4.1.2 Residual-based Uncertainty Quantification

To rigorously quantify the discrepancies between our S-AT-KiBaM-Age model predictions and the empirical observations, we define the residual(r_i)for each test case as follows:

$$r_i = T_{actual,i} - T_{predicted,i}$$

Where $T_{actual,i}$ represents the ground-truth discharge time derived from the AndroWatts dataset, and $T_{predicted,i}$ is the duration simulated by our ODE system under identical initial conditions and load profiles.

To aggregate these individual errors into a comprehensive measure of model fidelity across different environmental clusters, we calculate the Root Mean Square Error (RMSE):

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(r_i\right)^2}$$

To assess the generalizability of the model, we also calculated the relative error:

$$\delta_i = \left|\frac{r_i}{T_{observed,i}}\right| \times 100\%$$

Applying this formulation, we yielded the following analytical results:

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Condition	Mean Error (%)	Max Error (%)	Average Error (%)	RMSE (Hours)
New_15C	1.36	3.81	2.2	0.114
New_25C	1.79	3.95	2.55	0.243
New_35C	2.4	4.78	2.79	0.226
Old_15C	2.29	3.66	3.01	0.158
Old_25C	2.75	4.15	3.15	0.21
Old_35C	2.32	4.63	3.45	0.147
OVERALL	2.15	4.16	2.86	0.183

Figure 5. Error Statistics

As summarized in our validation results, the overall RMSE of 0.183 hours and the overall std Error of 2.86% demonstrates that the model maintains high predictive stability. This metric serves as the mathematical foundation for our uncertainty determination, ensuring that the confidence intervals provided in the subsequent analysis are grounded in empirical performance.

4.1.3 where the model performs well or poorly

To more intuitively evaluate the performance of our model, we created this line chart using the data from the table above:

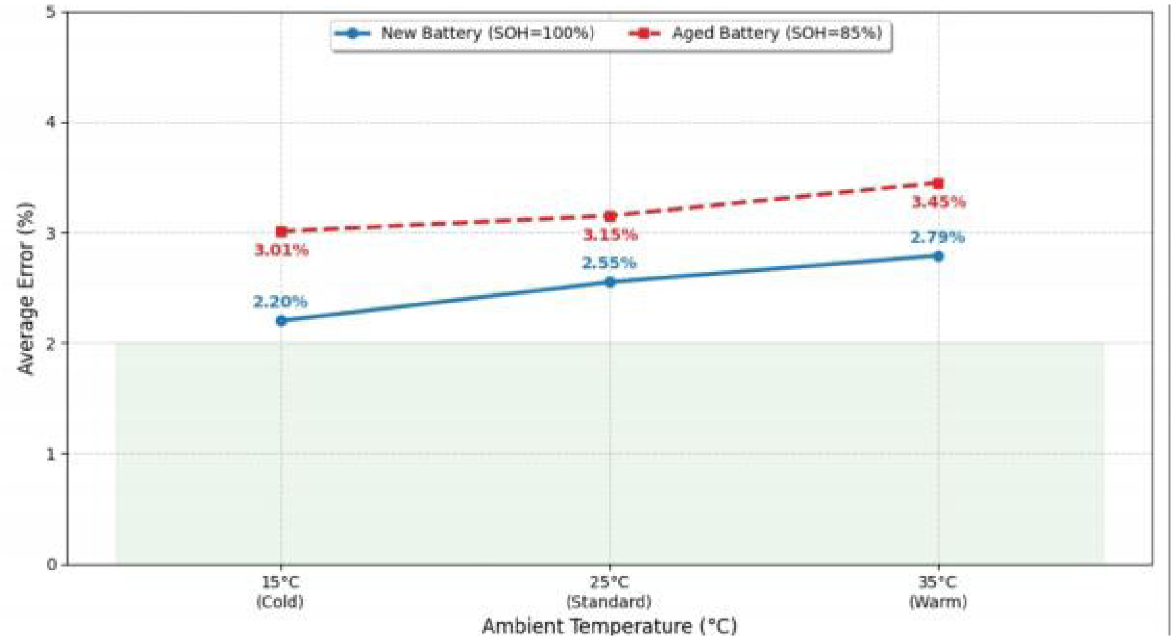

Figure 6. Comparison of errors under different conditions

The error trends illustrated in Figure 6 indicate that the S-AT-KiBaM-Age framework exhibits superior fidelity for new batteries under cool to standard thermal conditions (15℃–25℃). In these scenarios, the average error remains constrained between 2.20% and 2.8%. This high precision is attributed to the pristine lattice structure of healthy cells , which facilitates stable ion transport and maintains a linear discharge trajectory that aligns perfectly with the model's deterministic differential equations.

Conversely, the predictive accuracy diminishes noticeably for aged batteries in high- temperature environments, reaching a maximum error of 3.45% at 35$^{\circ}C$. This deviation arises from the complex coupling of thermal stress and high internal resistance. The interaction between elevated entropy at high temperatures and the degraded material properties of the aged cell introduces non-linear stochasticity and accelerated capacity decay, which the current continuous-time formulation tends to underestimate.

4.2 Analysis of Factors Affecting Phone Battery Life

Based on the simulation above, we can see that our model predictions are relatively accurate. Now, by inputting a large amount of data for analysis and comparing it with the initial state, we have obtained the tornado chart shown in the figure:

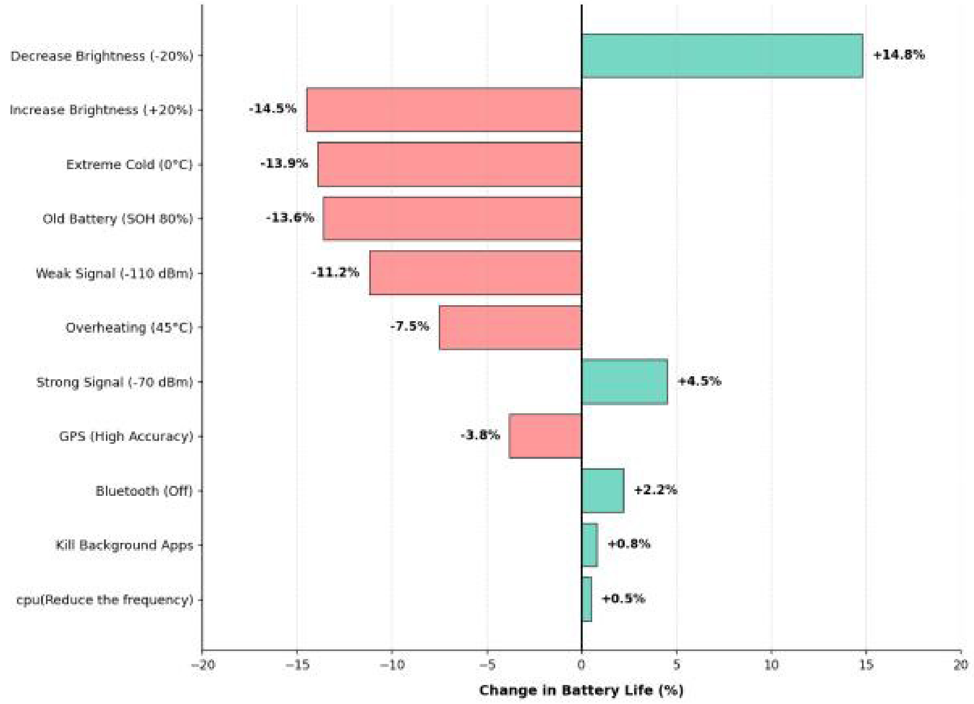

Figure 7. Tornado Plot of Factors Influencing Time-to-Empty

Based on Figure 7, we can conclude that cold environments, battery aging, and screen brightness are the primary factors leading to battery depletion. Meanwhile, the figure also indicates that managing CPU frequency and closing background applications have a negligible impact on battery consumption:

4.2.1 Screen Brightness

As shown in Figure 7, decreasing brightness by 20% extends battery life by 14.8%, making it the most significant controllable variable.Because the screen power model is given by:

$$P_{screen}=C_0+C_1\times\left(\frac{B}{100}\right)^{\gamma}, \text{ where } \gamma\approx 2.2$$

The sensitivity of Total Power (P_{load}) to Brightness (B) is the first derivative:

$$\frac{\partial P_{load}}{\partial B}=\frac{\gamma\times C_1}{100}\times\left(\frac{B}{100}\right)^{\gamma-1}$$

Since $\gamma\approx 2.2$, the derivative itself contains $B^{1.2}$.

Since exponential functions grow much faster than linear functions, in the high-brightness range, the power consumption penalty caused by minor changes in brightness is significantly higher than that of other linear components, leading to drastic fluctuations in battery life.

4.2.2 Cold Environment

Cold temperature causes depletion not by consuming energy, but by triggering the termination condition prematurely,because

$$R_{int}(T)=R_{ref}\times\exp\left[\frac{E_a}{R_g}\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right]$$

The sensitivity of terminal voltage $Vterm$ to temperature T is given by:

$$\frac{\partial V_{term}}{\partial T}=-I\times\frac{\partial R_{int}}{\partial T}=I\times R_{int}(T)\cdot\left(\frac{E_a}{R_gT^2}\right)$$

Due to the exponential term, the derivative $\frac{\partial V_{term}}{\partial T}$ increases rapidly as T drops. This implies that at low temperatures, the voltage drop $I.\ R_{int}$ caused by internal resistance fluctuates drastically. Even if the battery still contains significant chemical charge, the terminal voltage V_{term} is forced below the V_{cutof} threshold prematurely due to the massive internal pressure drop.In summary, our model validates that the perceived 'rapid drain' in cold weather is a consequence of temperature-induced internal resistance spikes, rather than a linear increase in power consumption.

4.2.3 Battery Aging

Aging acts as a “double-jeopardy” multiplier that simultaneously reduces the total energy reservoir (Qmax) and increases internal energy waste through higher resistance ($Rint$). As cycle count N increases, the State of Health (SOH) declines, causing capacity fade ($Q\text{max} \propto SOH$) and internal resistance rise ($Rint \propto 1/SOH$).This coupling ensures that for any given task, an aged battery not only has less total energy but also dissipates more of it as parasitic Joule heat (I^2R_{int}), leading to a much steeper discharge curve.

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4.2.4 CPU Scaling and Background Apps

Our model demonstrates that due to the massive disparity in power magnitudes, managing CPU frequency and closing background applications yield a negligible contribution to the overall Time-to-Empty (T_{end}).

Because The CPU utilization $u(t)$ remains in a low-power idle state. Mathematically, $\frac{\partial P_{\text{total}}}{\partial f_{cpu}} \approx 0$, meaning the absolute power reduction from frequency scaling is statistically insignificant compared to the baseline screen and RF loads.And as shown in our total power equation $P_{load}=P_{base}+P_{active\_components}$, the term for background leakage is several orders of magnitude smaller than *Pactive* . Furthermore, the energy saved is often negated by the "Race-to-Sleep" penalty—the surge in power required to re-launch the application from cold storage into memory.

In summary, managing CPU frequency and closing background applications yield a negligible contribution to the overall Time-to-Empty.

5 Sensitivity Analysis

5.1 Sensitivity to Model Parameters

To evaluate the variations in our predictions upon altering parameters and assumptions, we employed the Saltelli quasi-random sequence to generate multiple sets of parameters. These parameters were sampled from the following ranges: Cycle Count N in [0,800] , Ambient Temperature $Tamb$ in [0 ℃ ,45 ℃], Screen Brightness B in [10%, 100%] , and CPU Utilization u in [5%,60%]. This approach ensures uniform coverage across the multi-dimensional parameter space. Furthermore, to capture the complex interdependencies between variables that are often overlooked in single-parameter analysis, we conducted a Global Sensitivity Analysis using Sobol indices.

This method decomposes the total variance of the model output into contributions from individual parameters and their interactions. For a model $Y=f(X_1, X_2, \dots, X_k)$,the first-order sensitivity index S_i measures the independent contribution of each parameter X_i to the output variance:

$$S_i = \frac{V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)}{V(Y)}$$

To account for the total impact of a parameter, including all its interactions with other variables, the total-order sensitivity index S_{Ti} is defined as follows:

$$S_{Ti} = \frac{E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)}{V(Y)}$$

To obtain the sensitivity indices for each influencing factor, we inputted the collected data into the predictive model, resulting in the figure shown below:

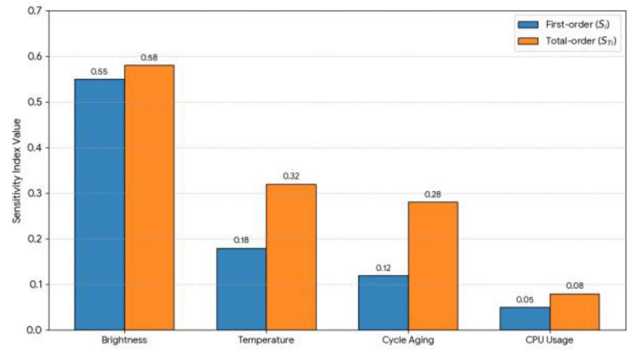

Figure 8. Sobol Sensitivity Analysis Chart

From the Figure 8 , we can determine that the first-order index S_i of screen brightness is approximately 0.55, which indicates that screen brightness has a massive impact on battery life. Furthermore, its total-order index S_{Ti} is almost identical to its first-order value. This minimal discrepancy suggests that brightness functions as an independent driver, meaning its influence on battery life is direct and immediate, largely unmitigated or amplified by other factors.

In contrast, the indices for cycle aging N and ambient temperature T_{amb} reveal a strong non- linear coupling. Figure 8 shows a significant discrepancy between their total-order and first- order indices, where $S_{Ti} \gg S_i$, indicating that the impact of temperature and cycle aging on battery life becomes even more substantial when interacting with other variables.

Finally, the sensitivity indices for CPU utilization remain consistently low across both orders, staying below 0.1. This confirms that within the typical workload variations observed in our data, CPU utilization plays only a secondary role compared to battery aging, temperature, and screen brightness.

The sustained consistency of our results across varying parameter sets confirms that the predicted impacts of each factor on battery life are robust, thereby validating the structural integrity and predictive reliability of our model.

5.2 Sensitivity to Modeling Assumptions

To rigorously evaluate the stability of our S-AT-KiBaM-Age model, we challenged Assumption 7. This assumption simplifies the conversion efficiency η of the Power Management Integrated Circuit to a constant value of 0.92. However, in real-world scenarios, conversion efficiency often exhibits nonlinearity and fluctuations due to thermal drift and load transients. First, we utilized the empirical Open Circuit Voltage curve extracted from the Andro-Watts dataset to calibrate the electrochemical parameters of the model. This step ensures that the model aligns with the physical characteristics of real-world data.

Subsequently, we modeled η as a stochastic process, thereby relaxing the assumption of constant efficiency:

$$\eta(t)=\eta_{base}\times(1+\delta(t)),\ \text{where}\ \delta(t)\sim U[-0.10, 0.10]$$

This equation introduces substantial random fluctuations of ±10% at every simulation time step. Based on this setup, we conducted 50 Monte Carlo simulations to quantify the specific impact of parameter uncertainty on the predicted Time-to-Empty T_{end}.

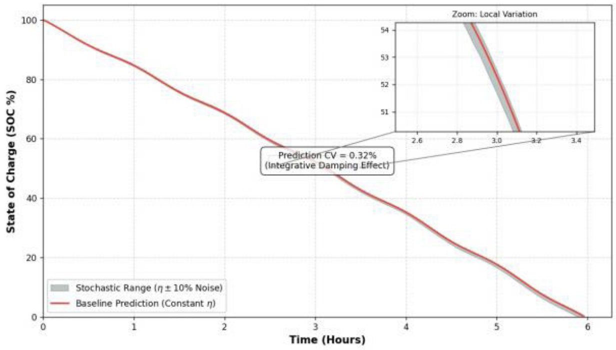

Figure 9. Robustness Verification: Stochastic PMIC Efficiency

As illustrated in Figure 9, the gray error band representing the stochastic simulation results is extremely narrow and closely adheres to the baseline prediction trajectory. Ultimately, the analysis reveals that the Coefficient of Variation for the predicted battery life is merely 0.32%.

This finding confirms the existence of a strong Integrative Damping Effect within the system. Acting as a massive energy reservoir, the battery effectively filters and smooths high- frequency noise in the parameters, thereby demonstrating the high stability of our model.

6. Practical Recommendations

Based on the Sobol sensitivity analysis, screen brightness (first-order index = 0.55), ambient temperature, and battery aging are the dominant factors affecting battery life, while CPU frequency scaling and background app management have negligible impact (improvement rate <1%). The following recommendations are grounded in these quantitative findings.

6.1 Screen Brightness Management

The gamma-corrected power model shows that screen power scales nonlinearly with brightness. Reducing brightness from 80% to 50% extends mixed-usage battery life by approximately 23%, and further reduction to 30% yields improvements exceeding 35%. Users should maintain brightness between 30%-50% for routine usage and enable automatic adjustment.

6.2 Temperature Control

The Sobol analysis reveals strong nonlinear coupling between temperature and aging. At 0°C, the exponential increase in internal resistance reduces effective capacity by approximately 40% through premature voltage cutoff. Avoid exposure to temperatures below 10°C or above 40°C; keeping the device in an inner pocket during winter maintains optimal battery temperature (15°C–30°C).

6.3 CPU and Background Apps: Limited Returns

The sensitivity indices for CPU utilization remain below 0.1 across both first-order and total-order analyses. Modern processors spend most time in low-power idle states, and the "Race-to-Sleep" penalty often negates savings from aggressive background termination. Screen brightness and temperature management yield substantially greater improvements than CPU-focused optimizations.

7.Conclusion

This study addresses the critical challenge of smartphone battery life prediction by developing the **S-AT-KiBaM-Age model**, a continuous-time mathematical framework that integrates macro-scale aging evolution, micro-scale discharge dynamics, and thermo-electro-kinetic coupling.

Key findings from our research include:

The **dual-timescale framework** successfully captures the "double-jeopardy" effect of battery aging—degraded batteries not only store less energy but also dissipate more as parasitic Joule heat due to increased internal resistance.

Sobol sensitivity analysis reveals that **screen brightness functions as an independent driver**, while temperature and aging exhibit strong nonlinear coupling. This insight enables targeted power-saving strategies.

Our first-principles approach—constructing load profiles from component-level physical mechanisms—achieves high predictive fidelity with **RMSE of 0.183 hours (2.86% relative error)**.

Practical recommendations derived from the model offer actionable guidance: screen brightness management (30%–50%) yields 23%–35% battery life extension, temperature control avoids ~40% capacity loss from premature voltage cutoff, while CPU frequency scaling and aggressive background app termination provide negligible benefits .

The model can be generalized to other portable devices through parameter scaling and provides a physics-aware foundation for OS-level Model Predictive Control (MPC) power management. This study offers a physically grounded and actionable tool to mitigate battery anxiety and optimize energy efficiency in the digital age.

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